

# Incentives Contracting

- ◆ A worker (risk neutral) is hired by a principal to do a task.
- ◆ Only the worker knows the effort she exerts (asymmetric information).
- ◆ The effort exerted affects the principal's payoff.
- ◆ The principal's problem: design an **incentives contract** that induces the worker to exert the amount of effort that maximizes the principal's payoff.

# Incentives Contracting

- ◆  $e$  is the agent's effort (not observable). Agent only has to fulfill one task.

- ◆ Principal's reward is  $y = f(e)$ .

(plus some external things, but we assume that average luck does not influence  $y$ )

- ◆ An incentive contract is a function  $s(y) = w \cdot y$  specifying the worker's payment when the principal's reward is  $y$ . The principal's profit is thus

$$\Pi_p = y - s(y) = f(e) - s(f(e)).$$

where  $p=1$

# Incentives Contracting

- ◆ Let  $\tilde{u}$  be the worker's (reservation) utility of not working.
- ◆ To get the worker's participation, the contract must offer the worker a utility of at least  $\tilde{u}$ .
- ◆ The worker's utility cost of an effort level  $e$  is  $c(e)$ , i.e. nobody likes to work.
- ◆  $c'(e) > 0$

# Incentives Contracting

So the principal's problem is choose  $e$  to

$$\max \Pi_p = f(e) - s(f(e))$$

subject to  $s(f(e)) - c(e) \geq \tilde{u}$ . (participation constraint)

i.e. the principal has to make sure that the agent will accept the wage offer. To do so he has to offer benefits (wage) that will cover worker's cost. To maximize his profit the principal designs the contract to provide the worker with her reservation utility level. That is, ...

# Incentives Contracting

the principal's problem is to

$$\max \Pi_p = f(e) - s(f(e))$$

subject to  $s(f(e)) - c(e) = \tilde{u}$ . (participation constraint)

Substitute for  $s(f(e))$  and solve

$$\max \Pi_p = f(e) - c(e) - \tilde{u}.$$

The principal's profit is maximized when

$$f'(e) = c'(e) \Rightarrow e = e^*.$$

# Incentives Contracting

The contract that maximizes the principal's profit insists upon the worker effort level  $e^*$  that equalizes the worker's marginal effort cost to the principal's marginal payoff from worker effort.

How can the principal induce the worker to choose  $e = e^*$ ?

# Incentives Contracting

- ◆  $e = e^*$  must be most preferred by the worker.
- ◆ So the contract  $s(y)$  must satisfy the **incentive-compatibility** constraint;

$$s(f(e^*)) - c(e^*) \geq s(f(e)) - c(e), \text{ for all } e \geq 0.$$

i.e. the contract should provide incentive for worker to choose higher effort. The agent will choose effort to maximize his own expected utility.

# Rental Contracting

- ◆ Examples of incentives contracts:
  - (i) **Rental contracts**: The principal keeps a lump-sum  $R$  for himself and the worker gets all profit above rental fee  $R$ :

$$s(f(e)) = f(e) - R.$$

- ◆ Why does this contract maximize the principal's profit?



# Rental Contracting

◆ Given the contract  $s(f(e)) = f(e) - R$   
the worker's payoff is

$$s(f(e)) - c(e) = f(e) - R - c(e)$$

and to maximize this the worker  
should choose the effort level for  
which  $f'(e) = c'(e)$ ; that is,  $e = e^*$ .

# Rental Contracting

- ◆ How large should be the principal's rental fee  $R$ ?
- ◆ The principal should extract as much rent as possible without causing the worker not to participate, so  $R$  should satisfy  $s(f(e^*)) - c(e^*) - R = \tilde{u}$ ;  
i.e.

$$R = s(f(e^*)) - c(e^*) - \tilde{u}.$$

# Other Incentives Contracts

- ◆ (ii) **Wages contracts**: In a wages contract the payment to the worker is

$$s(e) = we + K.$$

$w$  is the wage per unit of effort.

$K$  is a lump-sum payment.

- ◆  $w = f'(e^*)$  and  $K$  makes the worker just indifferent between participating and not participating.

# Other Incentives Contracts

- ◆ (iii) **Take-it-or-leave-it**: Choose  $e = e^*$  and be paid a lump-sum  $L$ , or choose  $e \neq e^*$  and be paid zero.
- ◆ The worker's utility from choosing  $e \neq e^*$  is  $-c(e)$ , so the worker will choose  $e = e^*$ .
- ◆  $L$  is chosen to make the worker indifferent between participating and not participating.

# Incentives Contracts in General

- ◆ The common feature of all efficient incentive contracts is that they make the worker the **full residual claimant** on profits.
- ◆ i.e. the last part of profit earned must accrue **entirely** to the worker.
- ◆ While this simple model gives a very nice result, there are all kind of circumstances where this result will not necessarily hold (eg: risk neutrality, the principal knows the cost function of the agent,  $y$  is perfectly observable).